your names_____

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PHY 321 Exercise 6: Earth's Orbit

Consider Earth as a point particle with mass, $m = 5.972 \times 10^{24}$ kg, in orbit around the sun, with mass $M = 1.998 \times 10^{30}$ kg. The potential energy is

$$V = -\frac{GMm}{r},$$

where $G = 6.674 \times 10^{-11}$ $\mathrm{m^3 kg^{-1} s^{-2}}$.

1. (5pts) Show that the units for G are consistent with the equation above if the masses are in kg, the radius is in m and the potential is in joules.

2. Write the equations of motion for x and y, i.e.

$$\frac{d^2x}{dt^2} = ...$$
$$\frac{d^2y}{dt^2} = ...$$

   in terms of x, y and GM.

3. (5 pts) The radius of Earth's orbit is 1.496×10^{11} m. If Earth's orbit is a circle, what is its speed, v_0?

4. (5 pts) The constant $GM = 1.333 \times 10^{20}$ $\mathrm{m^3 s^{-2}}$ using the values above. What is GM in units of $AU^3 \mathrm{years}^{-2}$? The symbol AU stands for a unit of length where the radius of Earth's orbit is 1.0 (known as astronomical units). What is v_0 in these units?

5. (5 pts) Write a program that calculates x, y, v_x and v_y as a function of t. Set the initial conditions as $x = R_0$, $y = 0$, $v_x = 0$, $v_y = v_0$, where R_0 is the radius of Earth's orbit and v_0 is the velocity of Earth's orbit. Use units of AU for distance and years for time.

6. (5 pts) Plot the trajectory, y vs. x in units of AU over a time period of 3 years.

7. (5 pts) Plot the trajectory again, but with v_0 set to half the value of Earth's orbit velocity.

8. (5 pts) Plot the kinetic energy, potential energy and total energy as a function of time for the last problem. Divide all the energies by m, i.e. plot E/m. Use units of $\mathrm{AU}^2/\mathrm{years}^2$.