your names____

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Exercise, Monday, Nov. 13,15, 2023

Here, we will define a function $L(x, \dot{x})$ in terms of the potential and kinetic energy of two particles,

$$\begin{aligned}
\mathcal{L} &\equiv T - V, \\
T &= \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2, \\
V &= V(x_1, x_2).
\end{aligned}$$

Consider the following integral

$$S \;\; = \int dt \mathcal{L}(\dot{x}_1, \dot{x}_2, x_1, x_2, t),$$

where $\dot{x}_i$ and x_i are functions of the time t.
If I minimize or maximize S (known as the action), the Euler-Lagrange equations are

$$\begin{aligned}
\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_1} &= \frac{\partial L}{\partial x_1}, \\
\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_2} &= \frac{\partial L}{\partial x_2},
\end{aligned}$$

For the problem below assume the two particles have the same mass, $m_1 = m_2 = m$, and that the potential is that of a single spring connecting the two masses.

$$V(x_1, x_2) = \frac{1}{2}k(x_1 - x_2)^2.$$

1. (5 pts) Using the Euler-Lagrange equations, find the equations of motion for both x_1 and x_2.
2. (5 pts) Write an equation of motion involving only $r \equiv x_1 - x_2$. (taking difference above equations)
3. (5 pts) Write an equation of motion involving only $R \equiv (x_1 + x_2)/2$. (summing two equations above)
4. (10 pts) Repeat (a-c), but assume the particles have different mass, and replace R with the center-of-mass coordinate, $R = (m_1x_1 + m_2x_2)/(m_1 + m_2)$. First, note that KE can be written as
   $$T = \frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu\dot{r}^2,$$
   where $\mu = m_1m_2/(m_1 + m_2)$. The potential energy is
   $$V = \frac{k}{2}r^2.$$
   Your goal is to write equations of motion for both R and r.
5. (5 pts) Show that the equations of motion from (4) reproduce those for (2) and (3) when the masses become equal.