Physics 321 Midterm #1 - Friday, Sep. 29, 2023

Some integrals:

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x),$$

$$\int \frac{dx}{1-x^2} = \tanh^{-1}(x),$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x),$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1}(x),$$

$$\int dx\ \tan(x+\phi) = -\ln(\cos(x+\phi))$$

$$\int dx\ \tanh(x+a) = \ln(\cosh(x+a)).$$

Write your name on EVERY page!

80 points possible, plus 25 points extra credit

---

1. A particle of mass m moves under the influence of a potential energy,

$$U(x) = \alpha|x|.$$

The particle starts at the origin moving with velocity v_0.

   (a) (5 pts) What is the particle's speed as a function of its position, x?

   (b) (15 pts) How much time, t, passes before the particle turns around?

your name_____

Extra Space for No.1

2. (15 pts) A mass m in a damped harmonic oscillator obeys the differential equation,

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0.$$

At a time $t = 0$ the particle is at a postion x_0 with zero velocity. Then, at $t = 0$, an additional force is added. The force is exponentially growing with time and has the form,
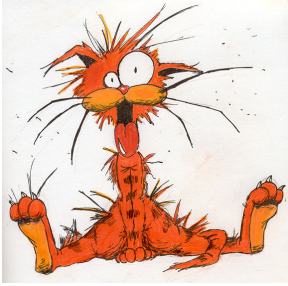
$$F(t) = F_0 e^{\lambda t}.$$

For large times, find the position as a function of time. Assume the time is so large that one can neglect any terms that die off exponentially with time. (You will NOT get full credit if you include the terms that die off!)

your name_____

Extra Space for No.2

3. Bill the cat, who has mass m, is dropped from a bridge with initial velocity $v_0 = 0$. The bridge is a height h above the water. The drag force on Bill has magnitude bAv^2, where A is Bill's cross sectional area and b is the drag coefficient. Give all answers in terms of b, A, m, h and the acceleration of gravity g.

(a) (5 pts) What is Bill's terminal velocity, v_t? (if he were dropped from an infinite height)

(b) (10 pts) Solve for Bill's velocity as a function of time. Give your answer in terms of g, v_t and t.

(c) (10 pts) When dropped from the bridge, how much time is required to strike the water? Give your answer in terms of v_t, g and h

(d) (5 pts) If Bill was fed heavy marbles, then dropped again, would Bill take more or less time to strike the water. Justify your answer.

Extra Space for No.3

4. Underdog, who has mass M_0 is flying through the air with velocity $V_0\hat{x}$ when he encounters a stream of bullets headed directly his way. Each bullet has mass m_b and velocity $-v_b\hat{x}$. Underdog, being the great and gallant canine that he is, swallows all the bullets without harm. The bullets were fired by a machine gun at rest, which fires at a rate R_0 bullets per second. For questions involving the time t, $t = 0$ refers to the time Underdog encounters the first bullet.

(a) (5 pts) How many bullets does Underdog swallow before his velocity is zero? (If he never stops, just say so)

(b) (10 pts) What is Underdog's mass, $M(t)$, as a function of his velocity, $V(t)$? Give answer in terms of M_0, m_b, V_0, v_b and $V(t)$.

(c) (5 pts, Extra Credit) If Underdog's velocity is $V(t)$, how many bullets does Underdog swallow per second? Give answer in terms of R_0, v_b and $V(t)$.

(d) (10 pts, Extra Credit) Write a differential equation for $V(t)$ based on momentum conservation. The equation should include $V(t)$ and dV/dt, plus the constants M_0, m_b, V_0, v_b and R_0.

(e) (10 pts, Extra Credit) Solve for $V(t)$ in terms of the constants above.

Extra Space for No.4