your name KEY

Physics 321 Midterm #2 - Wed. Nov. 21

FYI: For the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

the solutions are

$$
\begin{aligned}
x &= A_1 e^{-\beta t}\cos\omega' t + A_2 e^{-\beta t}\sin\omega' t \quad \omega' = \sqrt{\omega_0^2 - \beta^2} \quad \text{(under damped)}\\
x &= A e^{-\beta t} + B t e^{-\beta t}, \quad \text{(critically damped)}\\
x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_i = \beta \pm \sqrt{\beta^2 - \omega_0^2}, \quad \text{(over damped).}
\end{aligned}
$$

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your name____________________________________

1. A particle of mass m moves in a spherically spherically symmetric potential,

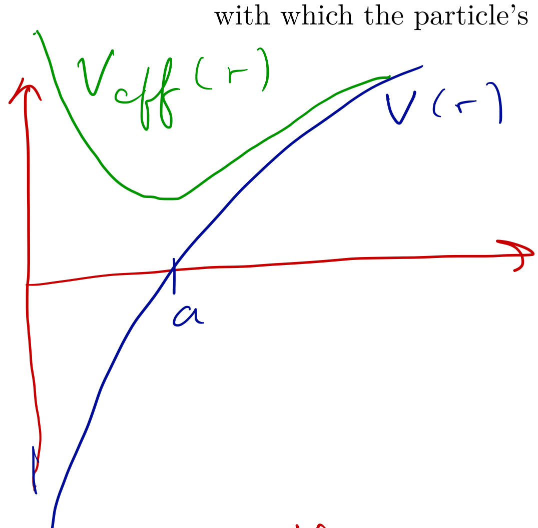
$$V(r) = V_0 \ln(r/a).$$

   (a) (5 pts) Sketch the potential $V(r)$ and, for a particle of angular momentum L, the effective potential for radial motion $V_{\text{eff}}(r)$.

   (b) (10 pts) If the particle is in a circular orbit of radius R, what is the angular frequency, $\dot{\theta}$, of the motion?

   (c) (10 pts) If the particle is given a slight radial push, what is the angular frequency, ω_r, with which the particle's radius oscillates about R?

a)



b) $|F| = \frac{V_0}{R} = m\dot{\theta}^2 R$, $\quad \dot{\theta} = \frac{\sqrt{V_0/m}}{R}$

c) $k_{eff} = \left.\frac{d^2 V_{eff}}{dr^2}\right|_R = \frac{3L^2}{mR^4} - \frac{V_0}{R^2}$

$L = mR^2\dot{\theta} = \sqrt{mV_0}\,R$

$k_{eff} = 3\frac{V_0}{R^2} - \frac{V_0}{R^2} = 2\frac{V_0}{R^2}$

$\omega_r = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{2V_0}{mR^2}} = \sqrt{2}\cdot\dot{\theta}$

your name____________________________________

2. A particle of mass m moves according to a spherically symmetric attractive potential,

$$V(r) = -\frac{\alpha}{r^2}.$$

If a particle reaches the origin, it is annihilated.

(a) (5 pts) Sketch the potential $V(r)$ and, for a particle of angular momentum L, the effective potential for radial motion $V_{\rm eff}(r)$.

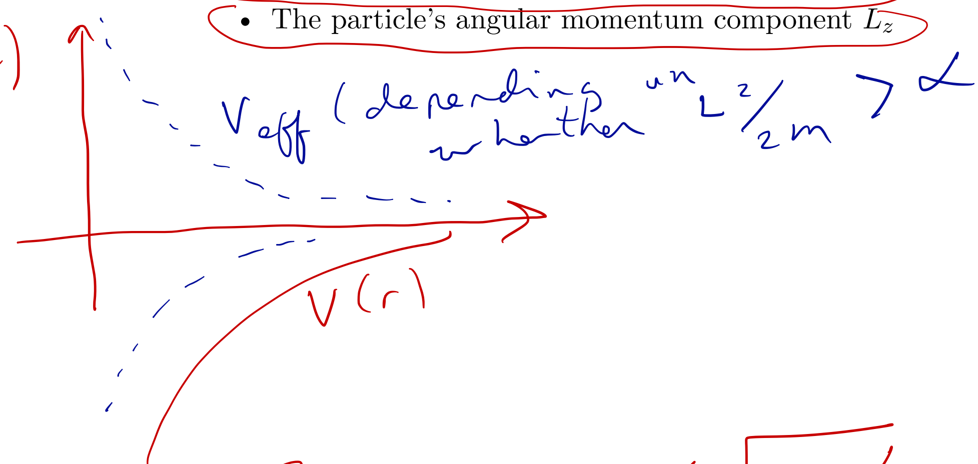
(b) (10 pts) If a trajectory has angular momentum L, what values of L lead to annihilation?

(c) (10 pts) For a beam of particles with energy E, what is the cross section for annihilation?

(d) (10 pts) Which of following quantities stay constant throughout the trajectory? Assume the initial impact parameter lies in the $\hat{y}$ direction and that the initial beam velocity is in the $\hat{x}$ direction. Circle the quantities that remain constant throughout the trajectory. (Note that $\hat{r}$ is a unit vector, $\hat{r} \neq \vec{r}$)

- The particle's kinetic energy
- The particle's potential energy
- The particle's total energy
- The particle's momentum component p_x
- The particle's momentum component p_y
- The particle's radial momentum $p_r = \hat{r} \cdot \vec{p}$
- The magnitude of the tangential momentum $p_t = |\hat{r} \times \vec{p}|$
- The particle's angular momentum component L_x
- The particle's angular momentum component L_y
- The particle's angular momentum component L_z

a)



b) $\frac{L^2}{2m} \overset{?}{>} \alpha$, $L < \sqrt{2m\alpha}$ gives annihilation

c) $L_{max} = m v b_{max}$, $b_{max} = \frac{\sqrt{2m\alpha}}{\sqrt{2mE}} = \sqrt{\frac{\alpha}{E}}$

$\sigma = \pi b_{max}^2 = \pi \alpha / E$

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Extra Work Space for #2

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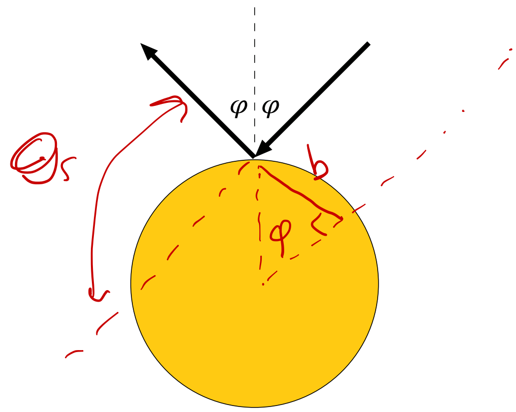
Physics 321 Midterm #2 - Wednesday, March 21



3. A beam of point particles of energy E and mass m are aimed at a spherical ball of radius R. The particles elastically bounce off the ball as shown in the figure.

   (a) (10 pts) Express the scattering angle θ_s in terms of the impact parameter b.

   (b) (15 pts) Express the differential cross section $d\sigma/d\Omega$ in terms of θ_s, R, E and m.

   (c) (5 pts) What is the total cross section σ?

   (d) (5 pts) A small detector is placed at 90 degrees from the incident beam in the azimuthal direction $\phi = 0$. The detector records all scattered particles with scattering angles $\pi/2 - \Delta\theta < \theta_s < \pi/2 + \Delta\theta$ and azimuthal angles $-\Delta\phi < \phi < \Delta\phi$. What is the cross section for scattering into this detector?

   (e) (5 pts) If a thin target has thickness t and if the number of scatterers (spherical balls) per unit volume is ρ, what fraction of incident beam particles scatter? Answer can be expressed in terms of σ, ρ and t.

   (f) (5 pts extra credit) If the target is not thin, what is the fraction of scattered particles?

a) $\theta_s = \pi - 2\varphi$, $b = R\sin\varphi$

$\theta_s = \pi - 2\sin^{-1}(b/R)$

b) $\sin\left(\frac{\pi - \theta_s}{2}\right) = b/R = \cos\theta_s/2$

$d\sigma = 2\pi b\, db = 2\pi R\cos\left(\frac{\theta_s}{2}\right) \cdot \frac{1}{2} R\sin\left(\frac{\theta_s}{2}\right) d\theta_s$

$= \frac{\pi}{2} R^2 \sin\theta_s\, d\theta_s = \frac{R^2}{4} d\Omega$

$\frac{d\sigma}{d\Omega} = R^2/4$

c) $\sigma = 4\pi \cdot \frac{d\sigma}{d\Omega} = \pi R^2$

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Extra Work Space for #3

(d) (5 pts) A small detector is placed at 90 degrees from the incident beam in the azimuthal direction $\phi = 0$. The detector records all scattered particles with scattering angles $\pi/2 - \Delta\theta < \theta_s < \pi/2 + \Delta\theta$ and azimuthal angles $-\Delta\phi < \phi < \Delta\phi$. What is the cross section for scattering into this detector?

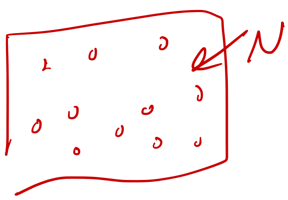
(e) (5 pts) If a thin target has thickness t and if the number of scatterers (spherical balls) per unit volume is ρ, what fraction of incident beam particles scatter? Answer can be expressed in terms of σ, ρ and t.

(f) (5 pts extra credit) If the target is not thin, what is the fraction of scattered particles?

d) $\frac{d\sigma}{d\Omega} = R^2/4$

$\Delta\Omega = \sin\theta_s \Delta\theta_s \cdot \Delta\varphi = \Delta\theta_s \Delta\varphi$

$\Delta\sigma = \frac{R^2}{4} \Delta\theta_s \Delta\varphi$

e) 

N frac $= N\sigma/A = \rho \cdot t \cdot \sigma$

f) Prob of not scattering

$= e^{-\rho\sigma t}$

Prob. of scattering

$= 1 - e^{-\rho\sigma t}$