Physics 321 Midterm #2 - Wednesday, Nov. 9, 2022

Some integrals:

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x),$$

$$\int \frac{dx}{1-x^2} = \tanh^{-1}(x),$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x),$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1}(x),$$

$$\int dx \ \tan(x+\phi) = -\ln(\cos(x+\phi))$$

$$\int dx \ \tanh(x+a) = \ln(\cosh(x+a)).$$

$$F(t) = \frac{f_0}{2} + \sum_n f_n \cos(n\omega t) + g_n \sin(n\omega t),$$

$$f_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} dt \ F(t) \cos(2n\pi t/\tau),$$

$$g_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} dt \ F(t) \sin(2n\pi t/\tau).$$

For the equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

The solutions are:

$$x = Ae^{i\omega t},$$

$$\omega = i\beta \pm \sqrt{\omega_0^2 - \beta^2},$$

$$x(t) = Ae^{-\beta t}e^{\pm i\sqrt{\omega_0^2-\beta^2}t}, \ \text{underdamped},$$

$$= Ae^{-\beta t \pm \sqrt{\beta^2-\omega_0^2}t}, \ \text{overdamped}.$$

Write your name on EVERY page!

80 points possible

1. (15 pts) A mass m is connected to a spring with spring constant $k = m\omega_0^2$ and has a very small damping. The mass also feels a periodic force

$$F(t) = F_0 \sin(\omega t),$$
$$\omega \neq \omega_0.$$

Assuming the force has been applied for a very very long time, find the displacement $x(t)$ for $t > 0$.

Extra Space for No.1

2. A mass m is connected to a spring with spring constant $k = m\omega_0^2$ and has an very small damping constant. It is subject to a periodic force,

$$F(t) = \begin{cases} -F_0, & -\tau/2 < t < 0, \\ F_0, & 0 < t < \tau/2 \end{cases},$$

$$F(t + \tau) = F(t).$$

(a) (15 pts) Expressing the force as a series

$$F(t) = \frac{f_0}{2} + \sum_n f_n \cos(n\omega t) + g_n \sin(n\omega t),$$

$$\omega = 2\pi/\tau,$$

what are the coefficients f_n and g_n? Note that $\omega \neq \omega_0$.

(b) (10 pts) Assuming the force has been active for a very very long time, express the displacement $x(t)$ in terms of the coefficients (as a sum) .

your name_____

Extra Space for No.2

3. Consider a particle of mass m and energy E which experiences an attractive spherically symmetric potential,

$$V(r) = -\frac{V_0}{r^2}.$$

If the particle reaches $r = 0$ it will annihilate.

(a) (5 pts) Sketch the effective (real plus centrifugal) potential for a particle with angular momentum L. Show potentials for a few values of L to demonstrate how V_{eff} depends on L.

(b) (15 pts) Calculate the cross section for annihilation.

Extra Space for No.3

4. (20 pts, no points awarded if answer has wrong dimension) You are an experimentalist planning a scattering experiment using a gold target and a much lighter projectile. You go online and find the mass density of gold, ρ_{Au}, and the mass of a gold atom, m_{Au}. Somebody sends you a target, but they didn't tell you the dimensions of the thin foil. You measure the width and length, W and L, then measure the mass of the foil, M, but you can't measure the thickness. The accelerator provides a beam, with N_{beam} particles incident on the foil over the duration of the experiment. One of your detectors is positioned at a scattering angle $\theta_s = 45°$. The detector has a cross-sectional area a and is placed a distance R from the center of the target. Over the six days you record N_{counts} in the detector. What is $d\sigma/d\Omega$ for $\theta_s = 45°$?

Extra Space for No.4