Nuclear Equation of State & Weak Interactions around neutrino-sphere

Gang Shen

Institute for Nuclear Theory

University of Washington

Main Collaborators: Sanjay Reddy (INT), Luke Roberts (Caltech); Chuck Horowitz (IUB), Evan O'Connor (CITA, Canada)

MSU Mini-Workshop "weak interactions, dense matter, and the r-process",
East Lansing, MI; Nov. 12-16, 2012

Outline

- Nuclear symmetry energy
- Nuclear equation of state (EOS)
- Neutrino interactions around neutrino-sphere
- Summary & Outlook

Nuclear Symmetry Energy in neutron rich matter

$$E(n_B, x_p) = E(n_B, x_p = 1/2) + E_{\rm sym}(n_B)\delta^2 + \cdots \qquad \delta = (1 - 2x_p)$$

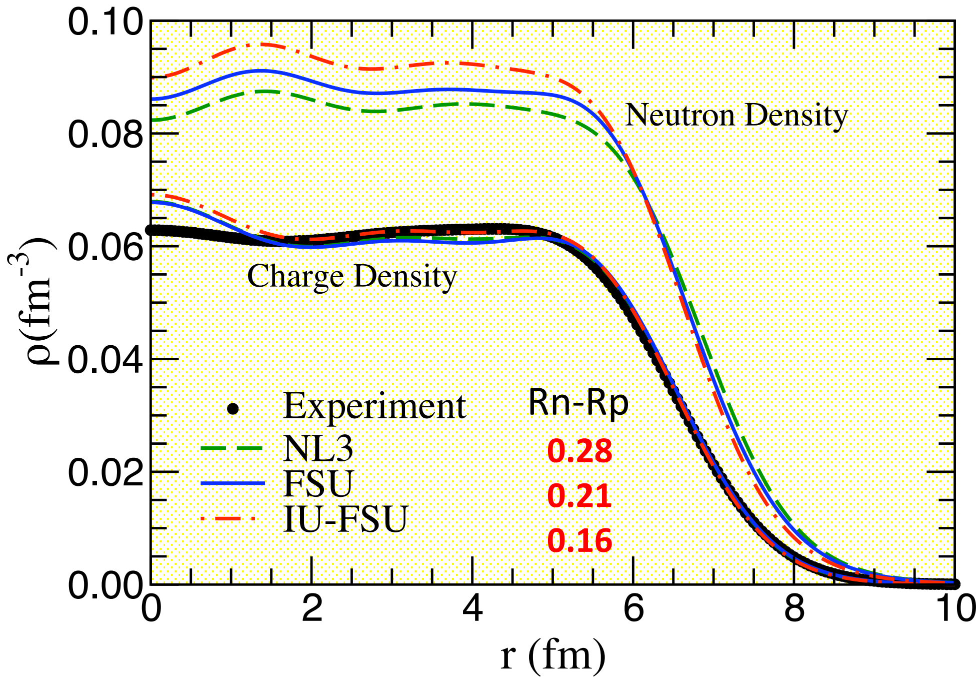
x_p : proton fraction

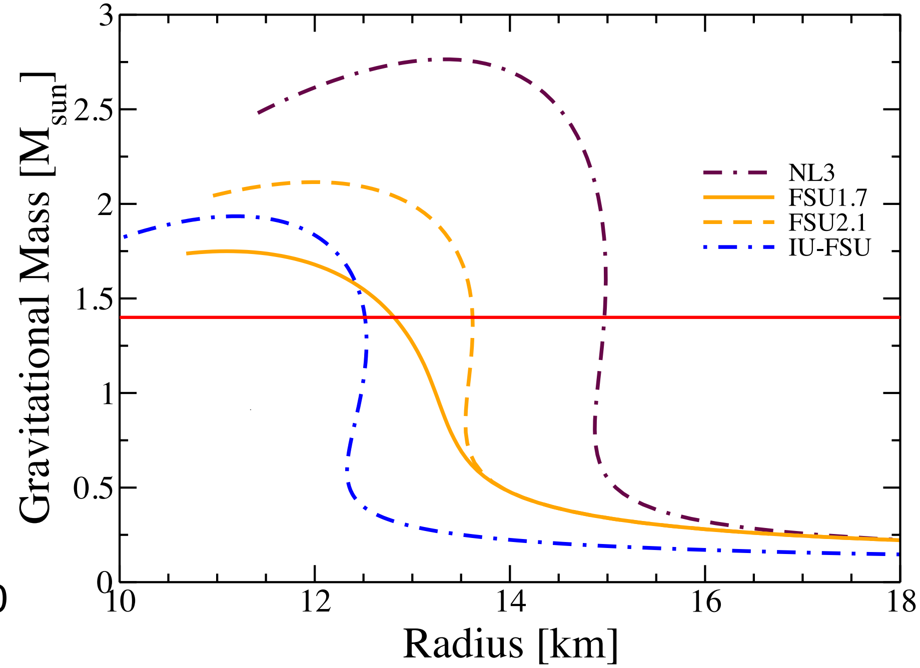
- E_{sym} describes how energy changes with p-n asymmetry
- Pressure ~ E'_{sym}=L/3, or large $E'_{sym}(n_0)$ – stiff EOS
- Large uncertainties in E_{sym} and E'_{sym} - reflected in diff. EOS

known correlations due to efforts in last decade:

- E'_{sym} ~ neutron radius of neutron rich nucleus, eg. Pb208
- E'_{sym} ~ neutron star radius
- E'_{sym} ~ Proto-Neutron Star (PNS) cooling *Talks by S. Reddy, L.. Roberts*
- E_{sym} ~ neutrino interactions *Talks by T. Fischer, L. Roberts*

Models	NL3	FSUGold	IU-FSU
$n_0 E'_{sym}(n_0)$ [MeV]	39	20	16





- A larger $E'_{sym}(n_0)$ indicates a bigger radius for 1.4 solar mass neutron star and a bigger neutron radius in 208Pb.

Fattoyev, Horowitz, Piekarewicz, GS (2010)

- Experiments/observations: PREX in JLab / model & data analysis on neutron star x-ray bursts/x-ray binary; (others, like isospin effects in heavy ion collision)
- Recent review: Lattimer & Lim (2012), Steiner & Gandolfi (2011)

Nuclear EOS in a unified framework

- One flexible framework for both nuclei and dense nuclear matter
- Ideal: density functional theory, for example, UNEDF
- Our choice
  - High density: relativistic mean field model for both nuclei and dense nuclear matter
  - Low density: virial expansion of nuclei and nucleons

GS, Horowitz, O'Connor (2011).
GS, Horowitz, Teige (2011).

High density: Relativistic Mean Field Theory

attractive — repulsive — Iso-vector

$$
\begin{aligned}
\mathscr{L}_{\rm int} \;=\; & \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi \\
& - \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 + \Lambda_{\rm v} g_\rho^2\,\mathbf{b}_\mu\cdot\mathbf{b}^\mu g_{\rm v}^2 V_\nu V^\nu
\end{aligned}
$$

- Ψ : Nucleon fields
- Meson/photon fields: classical expectation value ➔ mean field
- 7 adjustable parameters

EoMs in real space:

Nucleon:

$$\left[\alpha\cdot \boldsymbol{p} + V(\boldsymbol{r}) + \beta\left(M + S(\boldsymbol{r})\right)\right]\psi_i = \varepsilon_i\psi_i$$

Hartree Mean fields:

$$
\begin{aligned}
V(\mathbf{r}) &= \beta\{g_\omega \not{\omega}_\mu + g_\rho\vec{\tau}\cdot\vec{\not{\rho}}_\mu + e\frac{(1+\tau_3)}{2}\not{A}_\mu + \Sigma^R\}, \\
S(\mathbf{r}) &= \Gamma_\sigma\sigma,
\end{aligned}
$$

Mesons and Photons:

$$
\begin{cases}
\left(-\Delta + \partial_\sigma U(\sigma)\right)\sigma(\boldsymbol{r}) = -g_\sigma\rho_s(\boldsymbol{r}) \\
\left(-\Delta + m_\omega^2\right)\omega_0(\boldsymbol{r}) = g_\omega\rho_v(\boldsymbol{r}) \\
\left(-\Delta + m_\rho^2\right)\rho_0(\boldsymbol{r}) = g_\rho\rho_3(\boldsymbol{r}) \\
-\Delta A_0(\boldsymbol{r}) = e(\rho_c(\boldsymbol{r}) - \rho_e)
\end{cases}
$$

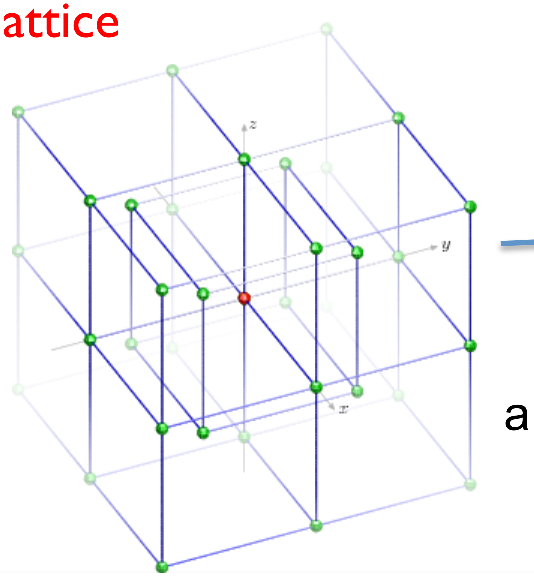
Finite temperature n_i: Fermi-Dirac stat.

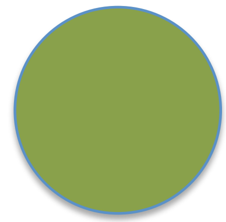
$$
\begin{cases}
\rho_s(\boldsymbol{r}) = \sum_{i=1}^{A}\bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r})n_i, \quad \rho_3(\boldsymbol{r}) = \sum_{i=1}^{A}\psi_i^+(\boldsymbol{r})\tau_3\psi_i(\boldsymbol{r})n_i \\
\rho_v(\boldsymbol{r}) = \sum_{i=1}^{A}\psi_i^+(\boldsymbol{r})\psi_i(\boldsymbol{r})n_i, \quad \rho_c(\boldsymbol{r}) = \sum_{i=1}^{A}\psi_i^+(\boldsymbol{r})\frac{1-\tau_3}{2}\psi_i(\boldsymbol{r})n_i
\end{cases}
$$

proto-neutron star crust (Γ>>1)

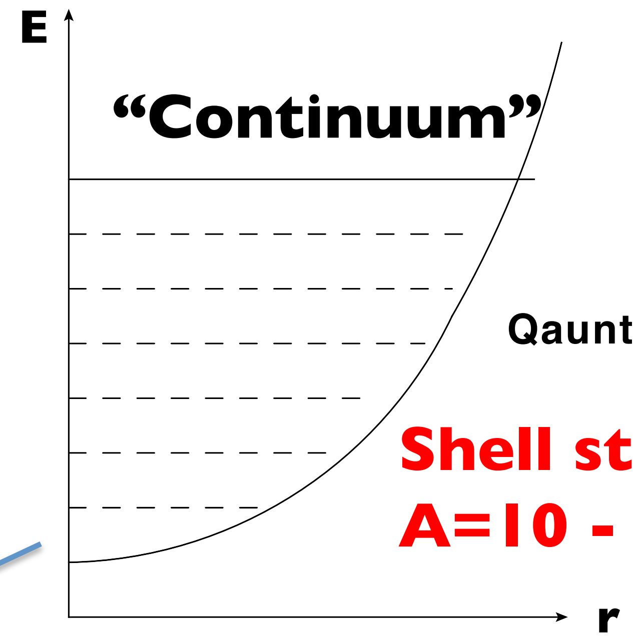
$$\Gamma = (Ze)^2/ak_BT$$

BCC lattice

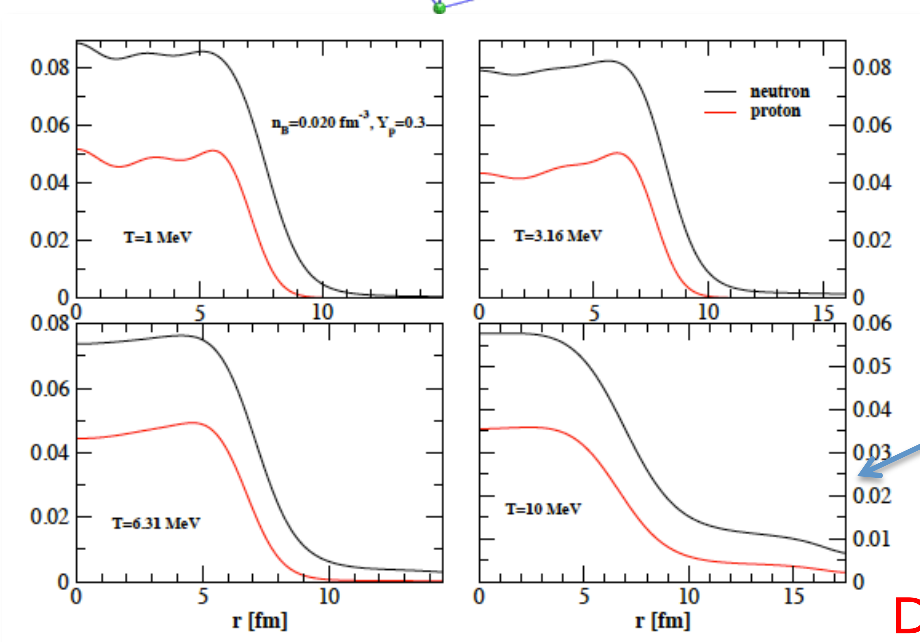




spherical Wigner-Seitz cell



Shell structure
A=10 - 1000



Density distribution in the unit lattice

Low density: Ensemble of nucleons and nuclei

- Grand partition function in virial expansion

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3}[z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}] + \frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha(z_n + z_p)b_{\alpha n}] + \sum_i \frac{1}{\lambda_i^3} z_i \Omega_i$$

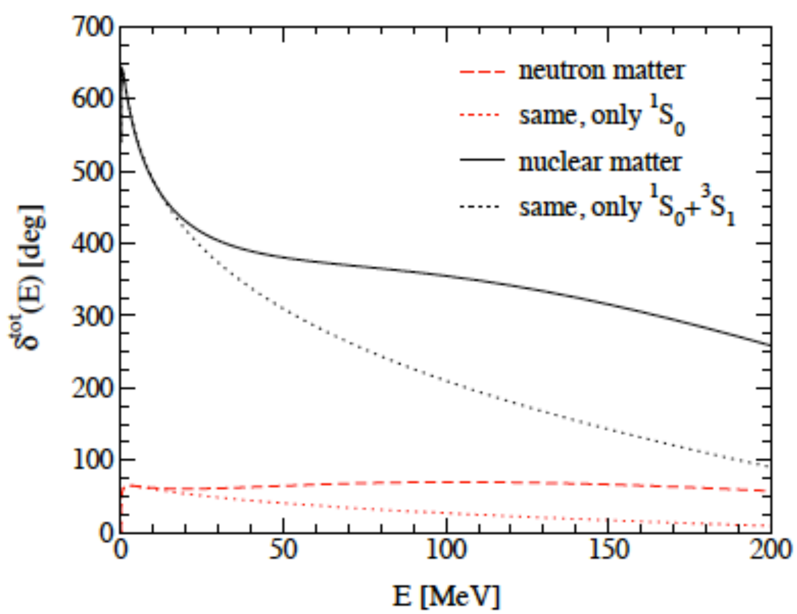
- nucleon-nucleon
- nucleon-alpha
- nuclei

1. Nucleon and alpha: mod. ind. Horowitz, Schwenk '05

2^{nd} virial coef. b_2 determined from elastic scattering phase shifts: model-independent

– b_n for n-n (p-p), – b_{pn} for n-p,
– b_α for alpha-alpha, – $b_{\alpha\text{-}n}$ for alpha-n

$$b_n(T) = \frac{1}{2^{1/2}\,\pi T}\int_0^\infty dE\, e^{-E/2T}\,\delta_n^{\rm tot}(E) - 2^{-5/2}\,.$$



Low density: Ensemble of nucleons and nuclei

- Grand partition function in virial expansion

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3}[z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}]$$
$$+\frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha(z_n + z_p)b_{\alpha n}]$$
$$+\sum_i \frac{1}{\lambda_i^3} z_i \Omega_i$$

➢ nucleon-nucleon

➢ nucleon-alpha

➢ nuclei

1. Nucleon and alpha: mod. ind. — Horowitz, Schwenk '05
2. Heavy species: 8980 nuclei — FRDM mass table: Moller et al '97. Δ_{rms} ~ 0.6 MeV

Chemical equilibrium ------ $\mu_i = Z\mu_p + N\mu_n \quad z_i = z_p^Z z_n^N e^{(E_i - E_i^C)/T}.$

Coulomb correction ------ $E_i^C = \frac{3}{5}\frac{Z_i^2\alpha}{r_A}[-\frac{3}{2}\frac{r_A}{r_i} + \frac{1}{2}(\frac{r_A}{r_i})^3]$

Nuclear partition function Ω_i ---- eg, Fowler, Engelbrecht, Woosley, '78

Nuclei spacing (r_i): $\frac{4}{3}\pi r_i^3(\sum_j Z_j n_j) = Z_i$

Low density: Ensemble of nucleons and nuclei

- Grand partition function in virial expansion

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3}[z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}]$$
$$+\frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha(z_n + z_p)b_{\alpha n}]$$
$$+\sum_i \frac{1}{\lambda_i^3} z_i \Omega_i$$

- ➢ nucleon-nucleon
- ➢ nucleon-alpha
- ➢ nuclei

1. Nucleon and alpha: mod. ind. — Horowitz, Schwenk '05
2. Heavy species: 8980 nuclei — FRDM mass table: Moller et al '97. Δ_{rms} ~ 0.6 MeV

Chemical equilibrium ------ $\mu_i = Z\mu_p + N\mu_n \quad z_i = z_p^Z z_n^N e^{(E_i - E_i^C)/T}.$

Coulomb correction ------ $E_i^C = \frac{3}{5}\frac{Z_i^2 \alpha}{r_A}[-\frac{3}{2}\frac{r_A}{r_i} + \frac{1}{2}(\frac{r_A}{r_i})^3]$

Nuclear partition function Ω_i ---- eg, Fowler, Engelbrecht, Woosley, '78

Nuclei spacing: $\frac{4}{3}\pi r_i^3 (\sum_j Z_j n_j) = Z_i$

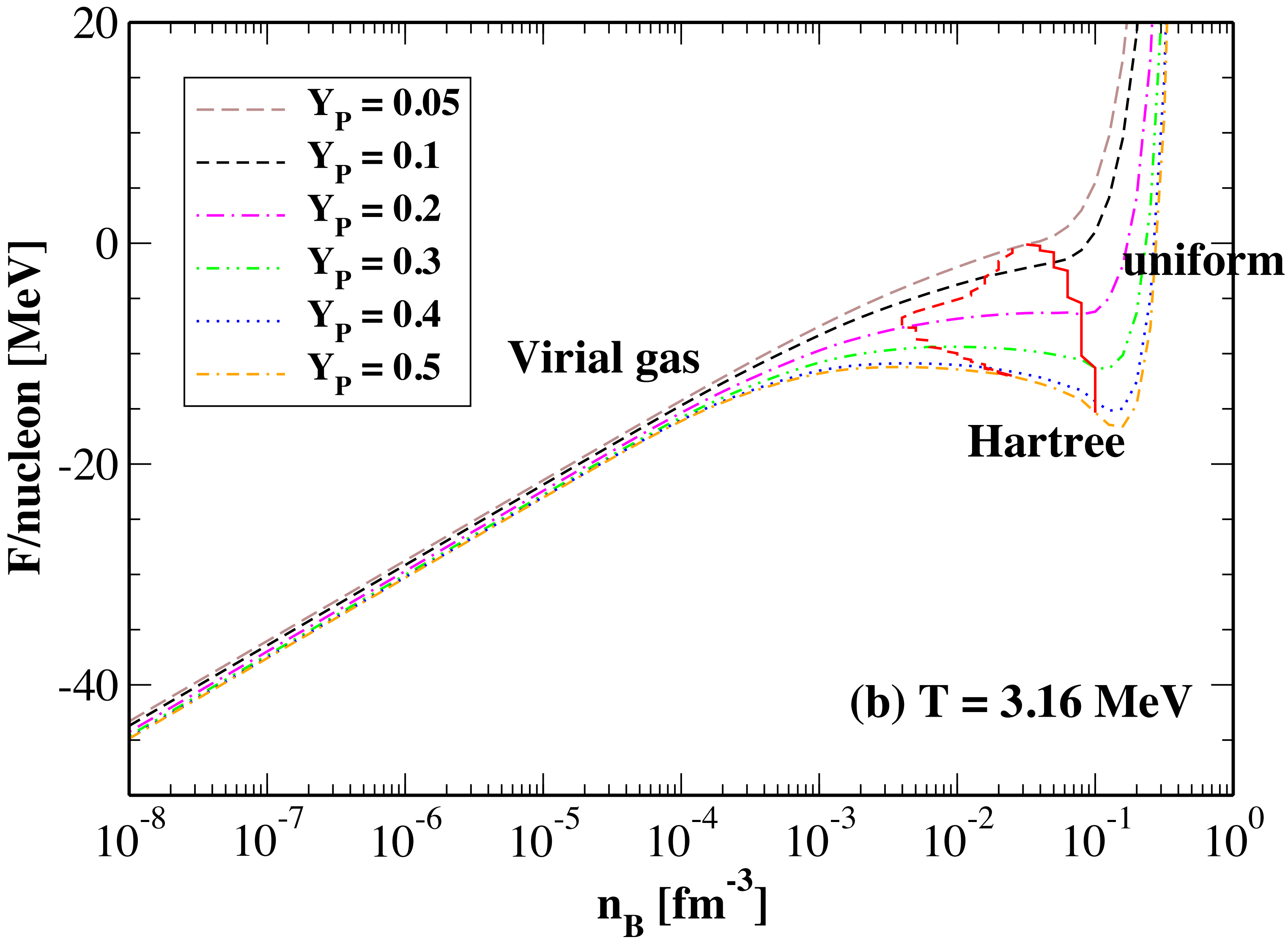
3. Solving: z_n, z_p, r_i:

$$n_B = n_n + n_p + 4n_\alpha + \sum_i A_i n_i$$ = Baryon

$$Y_P = (n_p + 2n_\alpha + \sum_i Z_i n_i)/n_B$$ = Charge

4. Mass fraction: $X_a = A_a n_a / n_B$

Matching EOS from low to high densities



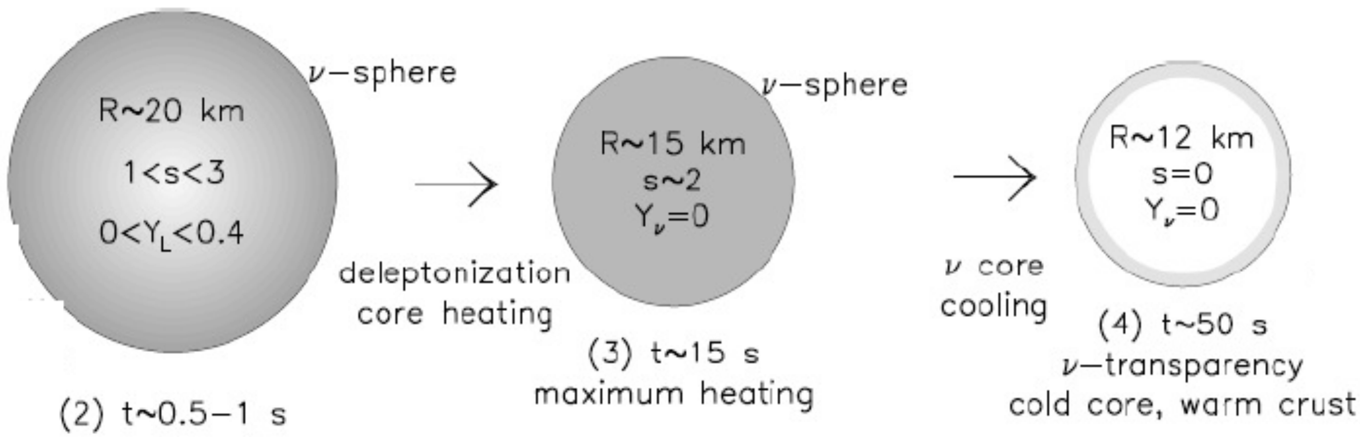
3-D Parameter spaces (T, ρ, Y_p) for gas-solid-liquid

	Virial Gas	Hartree	Uniform matter
Temperature [MeV]	0.1~20	0.1~12	0.1~80
Density [fm^{-3}]	1E-8~1E-1	1E-4~1E-1	1E-8~1.6
Proton fraction	0.05~0.56	0.05~0.56	0~0.56
# points in phase space	73,840	17,021	90,478
CPU time (hr)	10,000	100,000	100

- Matching them and interpolate to get thermodynamically consistent table.
- EOS tables: http://cecelia.physics.indiana.edu/gang_shen_eos/

Proto-neutron star (PNS) neutrinos and EOS

- Neutrino transport drives the evolution of a PNS (hot, lepton rich, SN remnant) → NS (cold, deleptonized, compact)



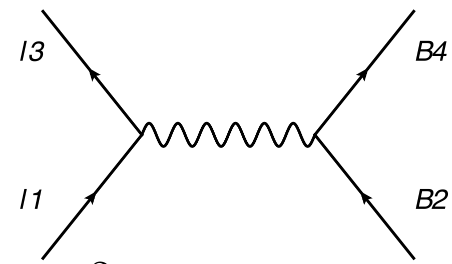
- Neutrino sphere: "effective boundary" for neutrinos to decouple; dynamical.
- Matter around neutrino-sphere important for spectra of "emitted" neutrino in neutrino driven wind

Neutrino-nucleon scattering

- Neutrinos couple to nuclear (isospin) density and (isospin) spin density:

V-A theory: $j^\mu(x) = \bar{\psi}(x)\gamma^\mu(C_V - C_A\gamma_5)(\tau_j)\psi(x)$

Non Relativistic limit: $\rightarrow C_V\psi^+(\tau_j)\psi\delta^{\mu 0} - C_A\psi^+\sigma^i(\tau_j)\psi\delta^{\mu i}$



$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta dE_3} = \frac{G_F^2}{4\pi^2}E_3^2(1-f_3(E_3)) \times[C_V^2(1+\cos\theta)S_\rho(q_0,q)+C_A^2(3-\cos\theta)S_\sigma(q_0,q)]$$

- Free Fermi gas response functions: peaked around $q_0 \sim 0$

$$S_{\rho,\sigma}(q_0,q) = \frac{1}{2\pi^2}\int d^3p_2\delta(q_0+E_2-E_4)f_2(1-f_4),$$

Dispersion $E_i(p) = M_i + p^2/2M_i$

Mean field (MF) shifts in charged current reactions

Roberts (2012), Roberts, Reddy, GS (2012)
Martinez-Pinedo et al. (2012)

- Dispersion relation of nucleon quasi-particle from MF EOS:

$$E_i(k) \;=\; \sqrt{k^2 + M^{*2}} + U_i,$$

$$\Delta U = U_n - U_p = \alpha * (n_n - n_p)$$ **~ symmetry energy**

- Response function is peaked around $q_0 \approx -\Delta U$, for $\nu_e + n \to p + e$

- Around ν-sphere, mean field shift is comparable to temperature → increases phase space in final state of electron exponentially !

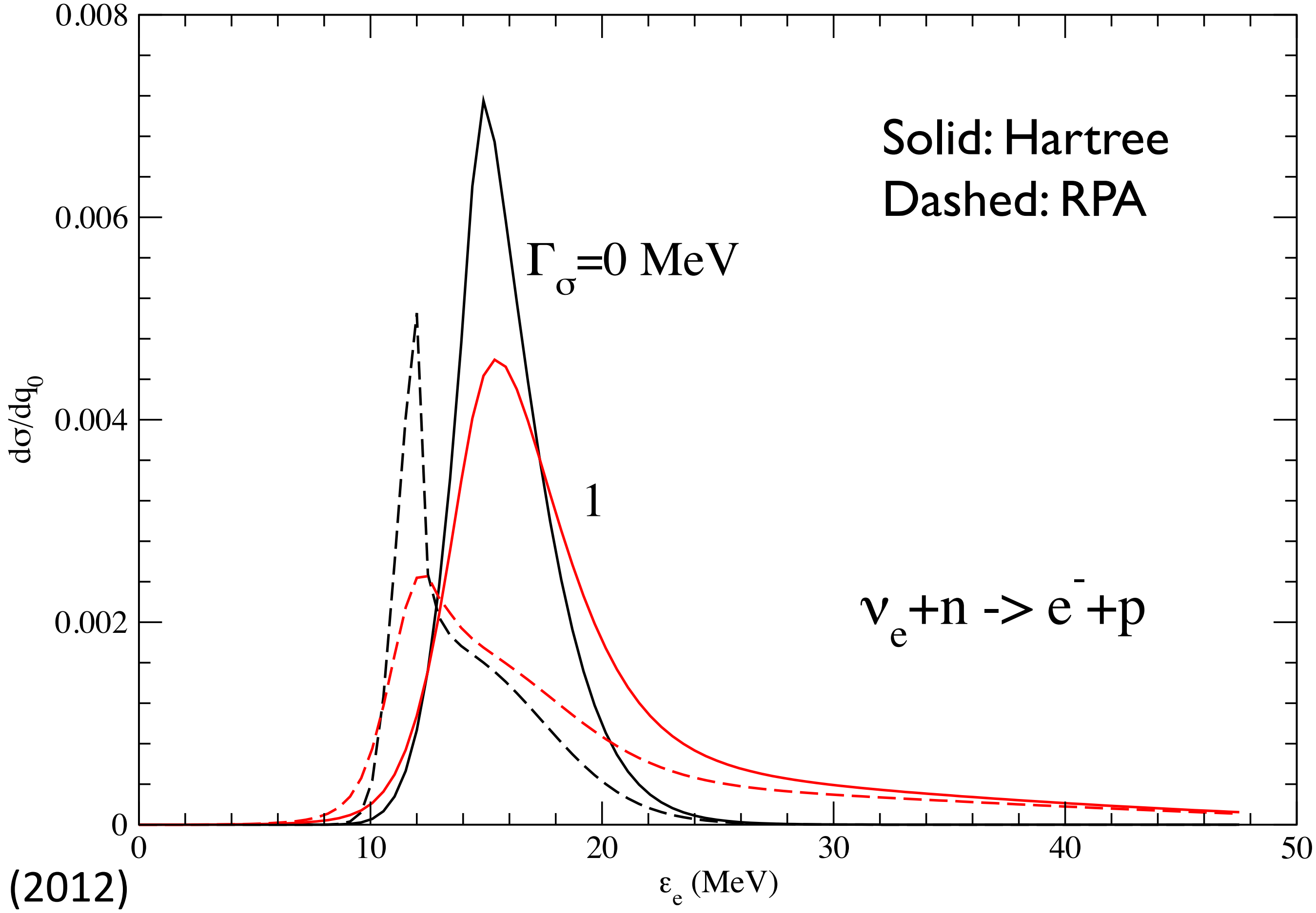
$$E_e = E_\nu + \Delta U \quad d\sigma \sim E_e^2 (1 - f_e(E_e))$$

- ν_e decouple at lower temp., while $\bar{\nu}_e$ at higher temp. May reduce electron fraction in neutrino driven wind – help r-p nucleosynthesis

$$Y_{e,\mathrm{NDW}} \approx \left[1 + \frac{\dot{N}_{\bar{\nu}_e} \langle \sigma(\epsilon)_{p,\bar{\nu}_e} \rangle}{\dot{N}_{\nu_e} \langle \sigma(\epsilon)_{n,\nu_e} \rangle}\right]^{-1}$$

Charged current reactions in RPA and beyond

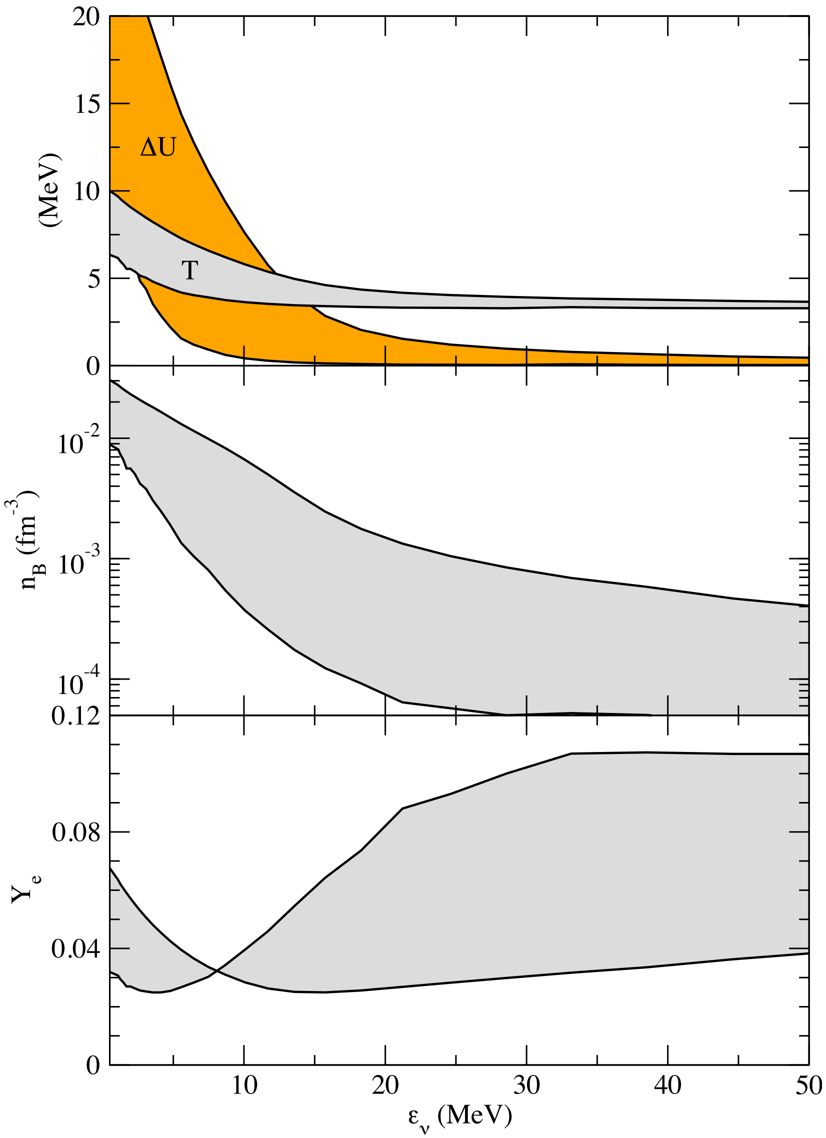
T=8 MeV
N=0.006 fm-3
Ye=0.03
μ_e=30 MeV
E_ν=12 MeV



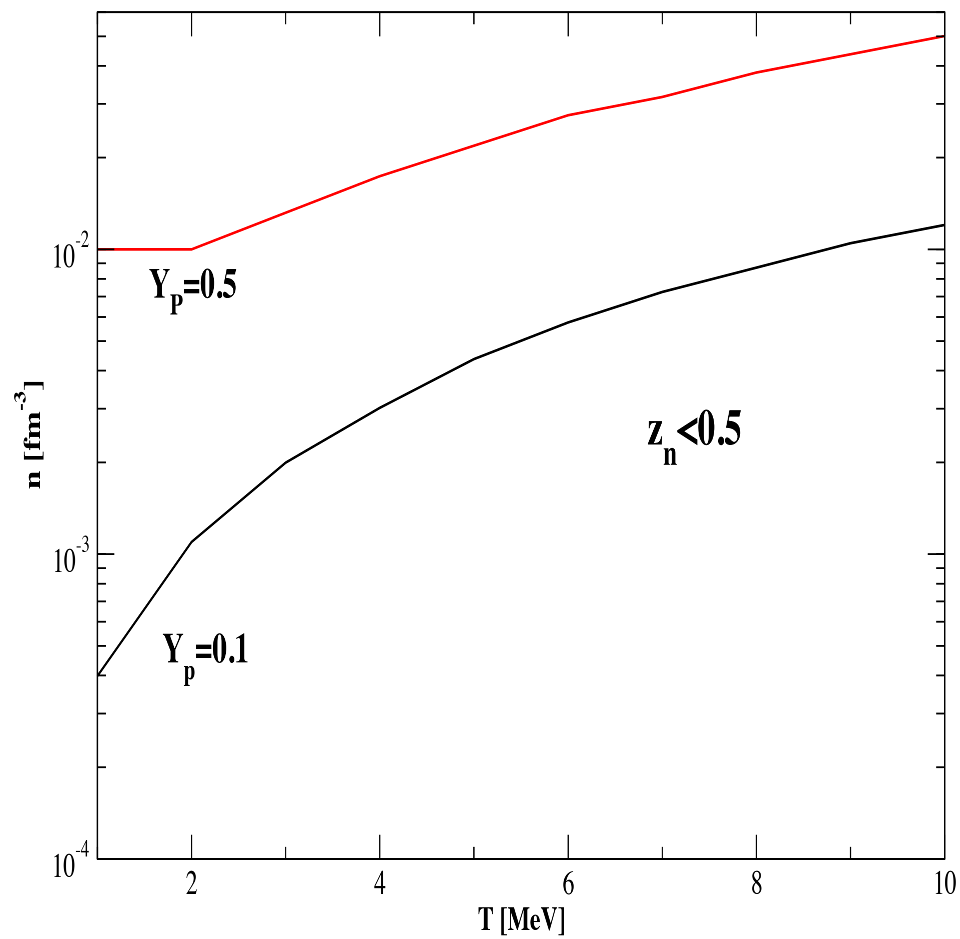
Roberts, Reddy, GS (2012)

- RPA suppresses response and shifts its strength via collective mode. Multi-particle dynamics enhances it.
- Net effect mild suppression. Follow SN trajectory to study details.

Typical conditions around nu-sphere



Courtesy: Luke Roberts



Virial expansion of n, p to 2nd order

Virial expansion for MF shifts

- Virial EOS of n, p: $$P = \frac{2T}{\lambda^3}\{z_n + z_p + (z_n^2 + z_p^2)b_n + 2z_p z_n b_{pn}\},$$
  - 2^{nd} virial coeff. b_n, b_{pn} from mod. ind. scattering phase shifts Horowitz, Schwenk '05
  - Suitable for matter around neutrino-sphere
- Single particle energy (MF) shift I, II:

I.

$$E_n = \left(\frac{\partial f}{\partial n_n}\right)_{n_p} = T\ln\frac{n_n\lambda^3}{2} - \lambda^3 T(n_n b_n + n_p b_{pn}), \quad (16)$$

$$E_p = \left(\frac{\partial f}{\partial n_p}\right)_{n_n} = T\ln\frac{n_p\lambda^3}{2} - \lambda^3 T(n_p b_n + n_n b_{pn}). \quad (17)$$

$$U_n = E_n - E_n^0 = -\lambda^3 T(n_n \hat{b}_n + n_p b_{pn}), \quad (19)$$

$$U_p = E_p - E_p^0 = -\lambda^3 T(n_p \hat{b}_n + n_n b_{pn}), \quad (20)$$

$$\Delta U = U_n - U_p = \lambda^3 T(n_n - n_p)(b_{pn} - \hat{b}_n)$$

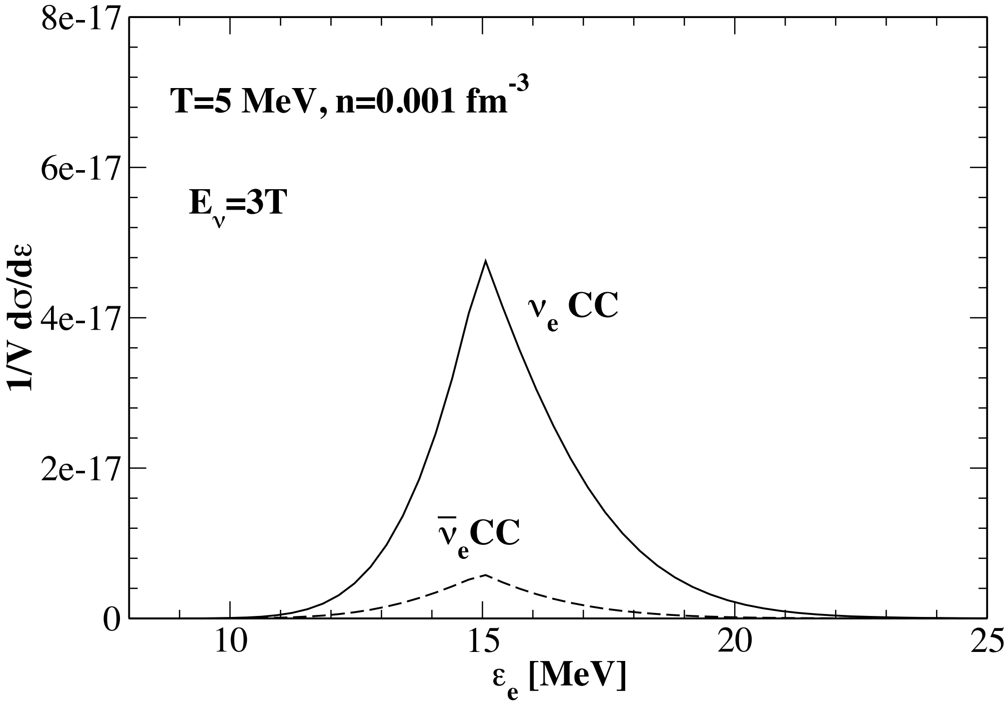
II.

$$U_n = \mu_n - \mu_n^f = -\lambda^3 T(n_n \hat{b}_n + n_p b_{pn}) + O(n_i^2).$$

Horowitz, GS, O'Connor, Ott, 2012

Virial expansion for MF shifts

- Virial EOS of n, p: $$P = \frac{2T}{\lambda^3}\{z_n + z_p + (z_n^2 + z_p^2)b_n + 2z_p z_n b_{pn}\},$$
  - 2^{nd} virial coeff. b_n, b_{pn} from mod. ind. scattering phase shifts Horowitz, Schwenk '05
  - Suitable for matter around neutrino-sphere
- Single particle energy (MF) shift:

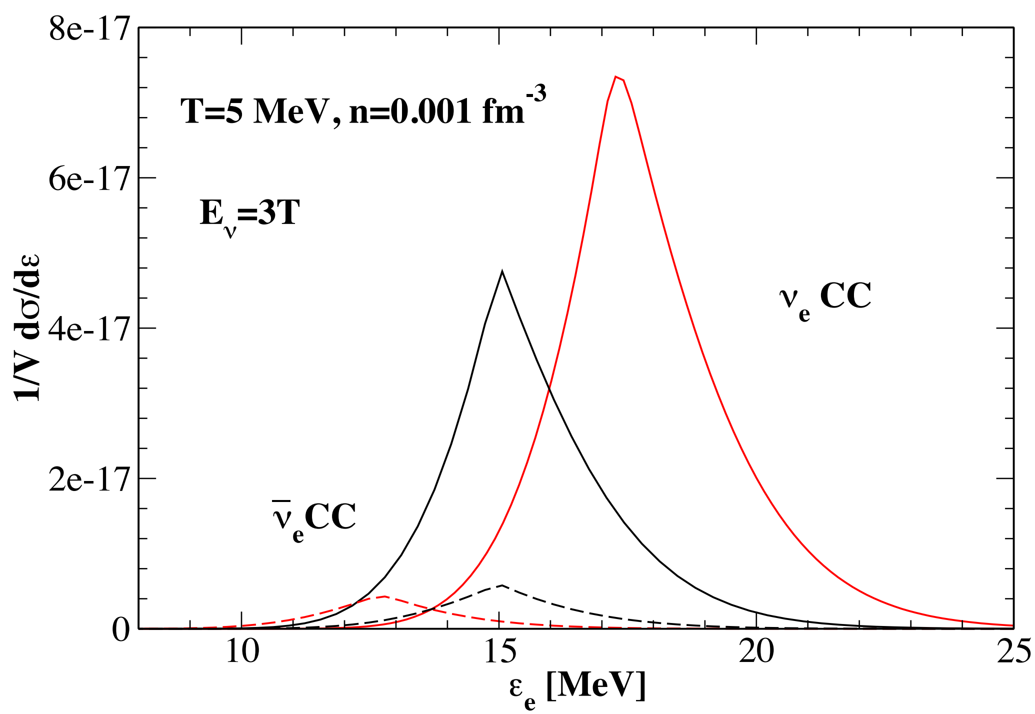


- W/O MF shift:
  dσ ~ density of n or p

Horowitz, GS, O'Connor, Ott, 2012

Virial expansion for MF shifts

- Virial EOS of n, p: $$P = \frac{2T}{\lambda^3}\{z_n + z_p + (z_n^2 + z_p^2)b_n + 2z_p z_n b_{pn}\},$$
  - 2^{nd} virial coeff. b_n, b_{pn} from mod. ind. scattering phase shifts Horowitz, Schwenk '05
  - Suitable for matter around neutrino-sphere
- Single particle energy (MF) shift:



- W/O MF shift:
  dσ ~ density of n or p
- W MF shift

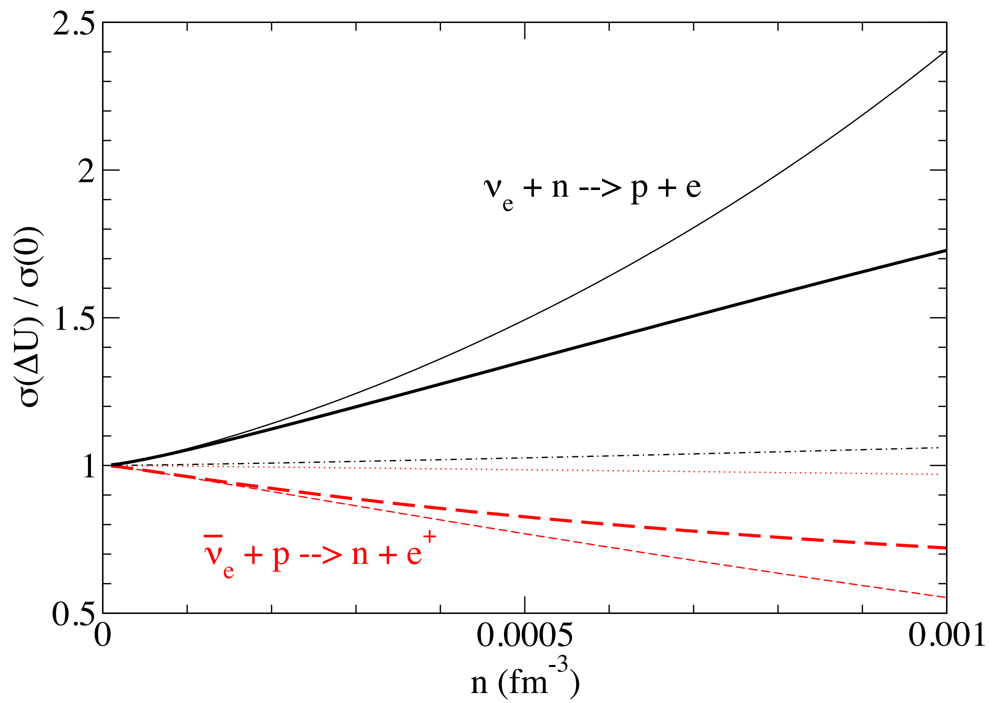
Horowitz, GS, O'Connor, Ott, 2012

Virial expansion and other MF models predictions

- ratio of cross sections

$$\frac{\sigma_{\nu_e}(\Delta U)}{\sigma_{\nu_e}(0)} = \frac{(E_\nu + \Delta U)^2[1 - f(E_\nu + \Delta U)]}{E_\nu^2[1 - f(E_\nu)]}. \qquad \frac{\sigma_{\bar{\nu}_e}(\Delta U)}{\sigma_{\bar{\nu}_e}(0)} = \frac{(E_{\bar{\nu}} - \Delta U)^2}{E_{\bar{\nu}}^2}\Theta(E_{\bar{\nu}} - \Delta U)$$

- effect larger than MF, due to n-p correlation
- May reduce electron fractions in neutrino driven wind compared to MF
- Feedback: larger Y_e in ν-sphere from larger E_{sym}
- Need consistent simulation with EOS and virial response



Z < 0.16

Summary & Outlook

- Unified framework for equations of state of nuclei and dense nuclear matter, for extreme astrophysics scenarios.
- Neutrino spectra from PNS receives medium correction, which is dependent on EOS (E_{sym}).

Thank you !